Diffusion Equation

Many physical phenomena, such as fluid flow, heat conduction and semiconductor growth include a diffusion process. The action of the diffusion is a continuous process that has and increasing effect with time and typically manifests itself as a spreading and smoothing of the original distribution. Let the full effect of the diffusion process be written in operator notation as follows:

$$Kf = g \tag{1}$$

where f is the original distribution, $K{:}X \rightarrow Y$ represents the operation of diffusion over the time interval and g the final distribution.

Physical problems that include diffusion processes, such as those stated, can often be modelled by parabolic partial differential equation[1] governing the concentration u, having the following general form

$$\frac{\partial u(x,t)}{\partial t} = D\nabla^2 u(x,t) + \text{first directional derivatives of } u \tag{2}$$

The first term on the right hand side of the equation is the diffusion term and the coefficient $D(\geq 0)$ represents the rate of diffusion. Note that D need not be constant, it may for example be a function of position or time. The rate of diffusion may also vary with u as in material-dependent diffusion, resulting in a non-linear equation with $K=K(f)$. Often the first term is written $\nabla \bullet (D\nabla u)$, but by product differentiation the equation would again take the form (2). In the terminology of equation (1), $u(\mathbf{x},0)=f(\mathbf{x})$ and $u(\mathbf{x},T)=g(\mathbf{x})$. (The symbols ∇ and $\nabla.$ represent the grad and div operators[2]. The symbol ∇^2 represents the Laplacian operator[3]).

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[1] Partial Differential Equations

[2] Vector Calculus: Grad, Div and Curl

[3] Laplace Operator or Laplacian